A MATHE MATICAL MODELLING TO DESCRIBE THE EFFECTS OF ALKALI-AGGREGATE REACTIONS IN CONCRETE STRUCTURES

B. Capra, J.P. Bournazel and M. Moranville-Regourd
Laboratoire de Mécanique et Technologie
ENS Cachan / CNRS / Université Paris VI
61 avenue du président Wilson
94235 Cachan Cedex
France

ABSTRACT

Physico-chemical processes, such as alkali-aggregate reaction can induce expansion, damage and cracking of concrete structures like dams. Maintenance and repair are costly on one hand and the prediction of service life is still difficult on the other. The literature is rich of papers which consider the chemistry of such a reaction, but only a few papers treat about the mechanical modelling of induced effects of AAR.

Based on chemical analysis and microstructure observations, a mechanical model for characterising the degradation of concrete using the thermodynamics of irreversible processes is presented. The state of concrete as a material is described by different variables. To justify the modelling, major hypotheses such as the value of relative humidity, temperature and the effect of an external applied stress are discussed.

The first numerical results are presented. They concern some experimental specimens altered by AAR. The results in term of expansion, strains, stresses and damage are discussed.

Keywords : *AAR, computation, damage, modelling, structure, thermodynamics*

INTRODUCTION

Dams built more than fifty years ago are suffering from deteriorations induced by Alkali-Aggregate Reaction (AAR), that impair durability and might also affect, the safety of installations. AAR induces concrete expansions and generally leads to loss of strength and cracking. Now, there is an increasing interest in the structural response due to AAR effects. The needs and the nature of potential interventions must be supported by the knowledge of the spatial distribution and intensity of swelling process. Fictitious thermal loading conditions are generally used to reproduce the state of deformations and stresses in a dam. This technique has severe limits, the aim of this paper is to present a new phenomenological modelling capable to describe the AAR induced effects in concrete structures. Some considerations on chemical aspects of the reactions must be analysed. AAR are autogeneous reactions, the particularity of these reactions is that they occur inhomogeneously, so only local sites are involved. Among theories of ASR we will choose the mechanisms published by Dent Glasser and Kataoka (Dent Glasser and Kataoka 1981).

MATHEMATICAL MODELLING

In order to allow structural computations, a global approach has to be used. From a phenomenological point of view, the principal parameters of the chemical reactions are the potential of reactivity (A), temperature (T) and relative humidity (H). Furthermore, from a structural point of view, the function of the stress (σ) is far from being explained. So, we can express the AAR free expansion by :

$$\varepsilon^{aar} = \varepsilon^{aar}(A, T, H, \sigma) \tag{1}$$

This sort of formulation leads to a homogeneous formulation and a probabilistic approach must be introduced. Using a random distribution of reactive sites, the first results obtained by Capra (Capra and Bournazel 1995) have shown interesting achievements. Unfortunately, such an approach needs a large number of simulations, with different random distribution. In the case of a site structure, this will be irrealistic due to the cost of calculation. So it is possible to have a new approach including probabilistic considerations but easier to use.

relation reaction/expansion

Diamond, Barneyback and Struble (1981) published curves of expansion versus time for two temperatures and, less usual, the percentage of alkalis reacted in function of time. The idea was to plot, for the two temperatures, the expansion versus the alkalis reacted. The results (fig. 1) show that temperature has only a little influence on the curve shape. In modelling the kinetics of reaction, we considered that the quantity of alkalis reacted was a variable representative of the evolution of the reaction.

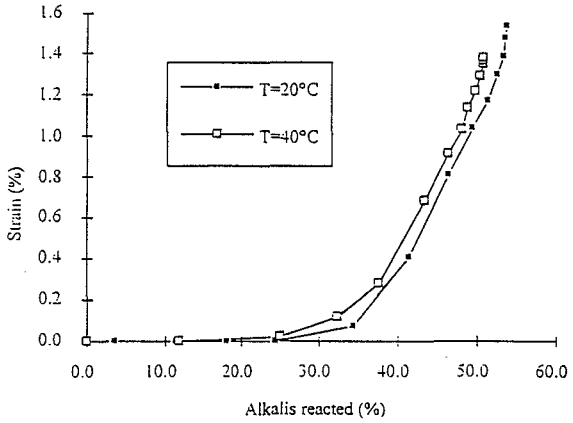


Fig 1 : Evolution of strain versus alkalis reacted

Supposing that the relation between expansion (ε^{aar}) and alkalis reacted (A) is bi-linear, we can write :

for $A < A_0$: $\varepsilon^{aar} = 0$ (2)

and

$$\text{for } A > A_0 : \varepsilon^{aar} = \frac{\varepsilon_o}{A_o}(A - A_o) \tag{3}$$

where ε_0 and A_0 are defined in figure 2

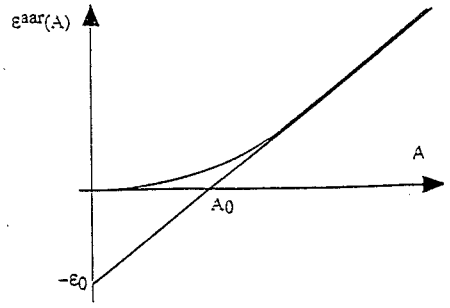


Fig 2 : Determination of the parameters

Then a relation between A and time, representing the kinetics of chemical reaction was introduced. AAR was expected to follow a first order kinetic law described by :

$$\frac{dA}{dt} = k_o . e^{-\frac{Ea}{RT}} . (1 - A) \tag{4}$$

This modelling led to evolutions, (fig. 3), which are in better agreement with experimental curves than the previous evolution law.

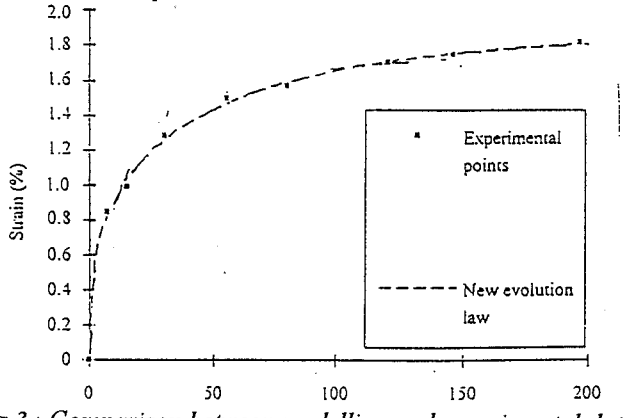


Fig 3 : Comparison between modelling and experimental data.

relation humidity/expansion

The moisture distribution in concrete is very important towards the reactions. Under approximatively 50%, there is enough water for the completion of reactions. Poole (Poole 1992) has published a curve showing the evolution of the expansion versus the relative humidity (Fig. 4). This curve can be represented by a power function :

$$\frac{\varepsilon^{aar}}{\varepsilon_o} = H^m \tag{5}$$

where H is the relative humidity and ε_0 the free expansion at H = 100%

Figure 4 shows a good agreement between the experimental data and mathematical modelling proposed.

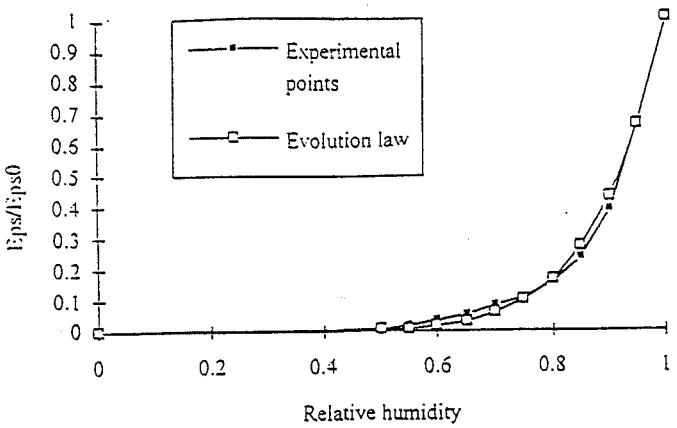


Fig 4 : Evolution of expansion versus relative humidity, comparison between experimental data and theoretical relation proposed.

relation stress / expansion

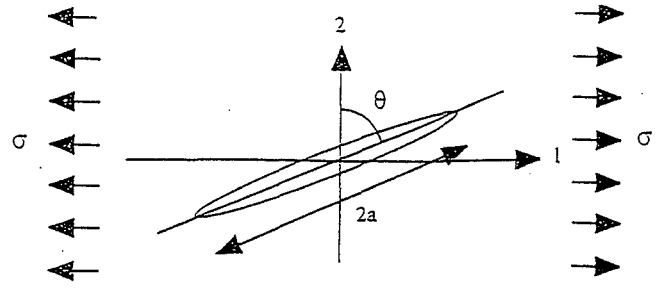
The role of two parameters has been studied (equations 2,3 and 5). Now, the most important has to be considered on a structural point of view : the role of stress on the expansion. Shayan and Quick (1992) have studied precast and prestressed concrete railway sleepers deteriorated by AAR. Although they were prestressed in the longitudinal direction, these structures were cracked parallel to the prestress. Therefore, we can see that an applied stress can reduce, by a structural effect, the expansion in this direction but not in the perpendicular one. Considering this, we have tried to reproduce these observations by the use of fracture mechanics. In order to model the expansion anisotropy, some hypotheses have been made. Cracks were considered to open only in mode I, and cracks interactions were not taken into account. The volume of gel created by AAR (Vg), was proportional to the free expansion (ε_o^{aar}). Therefore, this involved a state where structural expansions have started :

$$g(t) = \alpha_1 . \varepsilon_o^{aar}(t) \tag{6}$$

The local pressures (σ_{eq}) developed by the gel swelling are proportionnal to the volume of gel created : the more gel is, the more pressures are. According to the previous hypothesis, this leads to :

$$\sigma_{eq}(t) = \alpha_2 . Vg(t) = \beta . \varepsilon_o^{aar}(t) \tag{7}$$

So, we consider that the local pressure developed by the gel is proportionnal to the only global variable we can measure : the free expansion. Expansions will be created by cracks opening which is controled by fracture mechanics. In a bi-dimensionnal case, the stress intensity factor (K_I) is defined by (Fig. 5).



$$K_I = \sigma.\sqrt{\pi a}.\cos^2\theta$$

Fig 5 : Definition of the stress intensity factor

If a linear elastic medium is considered, in a bi-dimensional case, the resulting stress intensity factor is a function of the two principal stresses and the pressure inside the crack, this leads to :

$$\begin{aligned} K_I &= \sqrt{\pi a}.(\sigma_{eq}(t) + \sigma_1(t).\cos^2\theta + \sigma_2(t).\sin^2\theta) \\ &= \sqrt{\pi a}.(\beta.\varepsilon_o^{aar}(t) + \sigma_1(t).\cos^2\theta + \sigma_2(t).\sin^2\theta) \end{aligned} \quad (8)$$

The crack propagation, which is related to bulk deformation thus the expansion, is controlled by fracture mechanics statement : crack propagates if stress intensity factor is greater than the critical stress intensity factor (K_{IC}). Let $f(\theta,t)$ be fraction of cracks which propagates at time t. $\varepsilon^{aar}(\theta,t)$ is the strain at time t, in the direction θ, it is assumed that

$$\varepsilon^{aar}(\theta,t) = \varepsilon_o^{aar}(\infty).f(\theta,t) \quad (9)$$

Now, we have to characterize the initial cracking. This can be done by an image analysis, for example, which provides informations about the initial cracking distribution as a mean value and a standard deviation, as shown on fig. 6

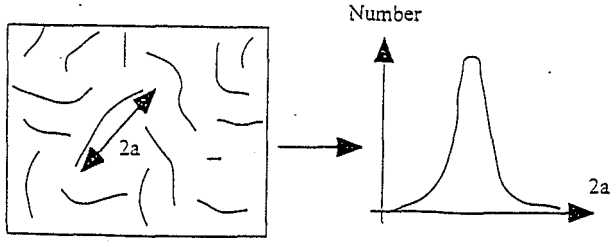


Fig 6 : Quantification of the initial cracking

Rather than a, the variable α defined by $\alpha = \sqrt{\pi}a$ is used, and its mean value $\bar{\alpha}$ and standard deviation $\underline{\alpha}$, which are deduced from the distribution of a. It is now necessary to join up f(θ,t), K_I and the crack distribution, let B(θ,t) :

$$B(\theta,t) = \frac{\left(\dfrac{K_{IC}}{\beta\varepsilon_o^{aar}(t) + \sigma_1(t).\cos^2\theta + \sigma_2(t).\sin^2\theta}\right) - \bar{\alpha}}{\underline{\alpha}} \tag{10}$$

and then, f(θ,t) takes the form :

$$f(\theta,t) = \int_{-\infty}^{B} e^{-\frac{u^2}{2}} du \tag{11}$$

Then, the expansion in the principal directions can be expressed by projection of the contribution of $\varepsilon^{aar}(\theta,t)$ for each θ in the principal axes, this leads to the expressions (12) and (13) :

$$\varepsilon_1^{aar} = \frac{\int_0^{\frac{\pi}{2}} \varepsilon^{aar}(\theta,t).\cos\theta.d\theta}{\int_0^{\frac{\pi}{2}} \cos\theta.d\theta} \tag{12}$$

and

$$\varepsilon_2^{aar} = \frac{\int_0^{\frac{\pi}{2}} \varepsilon^{aar}(\theta,t).\sin\theta.d\theta}{\int_0^{\frac{\pi}{2}} \sin\theta.d\theta} \tag{13}$$

This work has been done for a two dimensional problem but it can easily be extended to a three dimensionnal one, considering an ellipsoidal inclusion for the crack and projection on the three axes.

The evolution of the two principal strains ε_1 and ε_2, in a uniaxial compression σ_1, is presented on figure 7. There is a great reduction of the strain in the direction of the load, but in the perpendicular direction, the reduction is less important. Finally, this relatively simple way of description of the anisotropic expansion shows remarkable results. Nevertheless, it is necessary to have an experimental validation. That is why, in the same time, some experiments on concrete bars with uniaxial applied stress have been started. This could provide some informations about the effect of an applied stress on the diminution of the expansion.

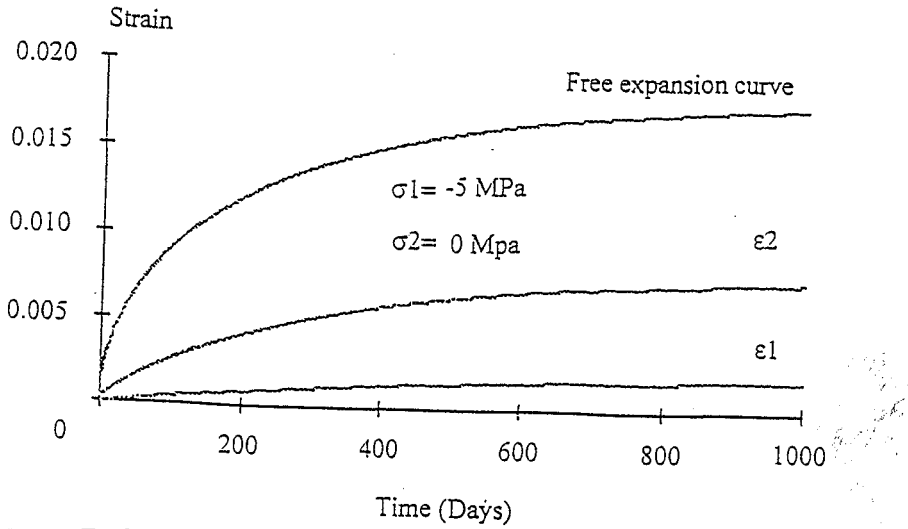


Fig 7 : Evolution of the free expansion, ε_1 and ε_2, in a uniaxial compression case versus time

NUMERICAL SIMULATIONS OF AAR

The previous paragraph allows us to bring to the fore the influence of the principal parameters of the AAR. Now, in the aim of numerical simulations on a real structure, we have to take into account all the previous considerations. Then, the following equation for the evolution of the irreversible strains due to AAR is proposed :

$$\varepsilon^{aar}(H,T,\tilde{\sigma},t) = (H)^m . \frac{\varepsilon_o}{A_o} . (1 - A_o - e^{-k_o . e^{-\frac{Ea}{RT}} . t}) . f(\tilde{\sigma}) \qquad (14)$$

where ε^{aar} and $\tilde{\sigma}$ are the strain tensor due to AAR and the stress tensor, $f(\tilde{\sigma})$ is the relation between stress and strain described in the previous paragraph. So, this function takes into account the principal parameters of AAR. Furthemore, this inelastic and anisotropic part will be coupled with a damageable concrete behaviour in attemping to numerical simulations. Previously, it will be necessary to make a thermal computation and one giving the repartition of the relative humidity H, in order to calculate $\varepsilon^{aar}(H,T,\tilde{\sigma},t)$.

CONCLUSIONS

Until now, AAR has brought about a lot of problems to engineers. It is difficult to take it into account with a good agreement towards reality because of the complexity of the phenomena. We have proposed here a new approach of modelling which takes into account some major parameters of the reactions. Such an approach is interesting on a theoretical point of view, because it allows with a relatively simple way, to model the anisotropy of the reaction. On a structural point of view, this anisotropy seems to be a

very important parameter. So, in order to make some simulations, this approach has to be validated by some experiments or in situ observations.

References

Capra, B. & Bournazel, J.P. 1995 "Perspectives Nouvelles pour la Prise en Compte des Alcali-Reaction dans le Calcul des Structures" *Materials and Structures*, 28, 71-73

Dent Glasser, L.S. & Kataoka, N. 1981 "The chemistry of Alkali Aggregate Reaction" Proc. of 5th Int. Conf. on AAR, Cape Town, Paper S 252/23.

Diamond, S., Barneyback, R.S. & Struble, L.J. 1981 "On the Physics and Chemistry of Alkali-Silica Reaction" Proc. of 5th ICAAR, Cape Town, Paper S 252/22

Poole, A.B. 1992 "Introduction to Alkali Aggregate Reaction in Concrete", *The Alkali-silica reaction in concrete*, ed by R.N. Swamy, Blackie, London

Shayan, A. & Quick, G.W. 1992 "Microscopic Features of Cracked and Uncracked Concrete Railway Sleepers", *ACI Materials Journal*, vol. 89, n°4, 348-361