MODELLING OF THE STRUCTURAL BEHAVIOUR OF AAR AFFECTED REINFORCED CONCRETE MEMBERS

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ABSTRACT

A non-linear two dimensional finite element analysis has been developed in order to assist in the understanding of the structural effects of alkali aggregate reaction (AAR) on concrete members. It has been observed that the application of relatively small compressive stresses to concrete during the development of the AAR can considerably inhibit expansion. This phenomenon has been shown to be important and has been included in the analysis as have the changes in material properties due to AAR. A brief description of the analytical model is given and experimental results from two tests are compared with the predictions of the analysis.
Key Words: Alkali aggregate reaction; finite element analysis.

INTRODUCTION

It has been observed that relatively small compressive stresses can have a considerable effect on the expansion of AAR specimens. Stresses of about 4 N/mm^2 can stop the expansion occurring almost completely. Figure 1 shows a typical relationship between expansion and stress (*Cope et al 1994).* To carry out any sensible analyses of AAR affected structures, this phenomenon needs to be included, in addition to the changes in material properties due to the development of AAR. A brief description of such an analysis follows, together with two examples of the use of the analysis.

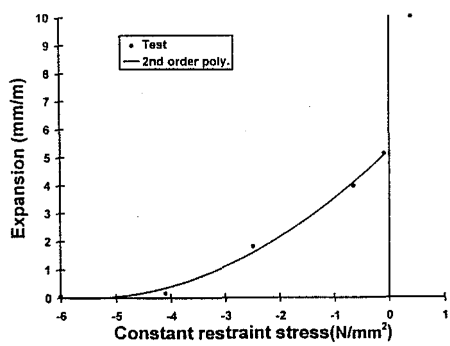

Fig. 1 Constant stress-expansion relationships

ANALYTICAL MODEL

A two dimensional, plane stress, finite element analysis, which includes material non-linearities, has been developed. This is capable of analysing members subjected to AAR and a set of external loadings, which can be varied or held constant throughout the development of the AAR. After the AAR is complete, the member can then be analysed up to failure, with a pattern of loading which can be different to that applied during the AAR.

The analysis can be considered to comprise three stages:

1. Analysis of the member under external loading.
2. AAR expansion analysis under the external loading.
3. Analysis of the affected member up to failure.

The first stage comprises a smeared crack non-linear finite element analysis for unaffected concrete. The third stage is similar but uses the deteriorating properties of the affected concrete as determined during stage 2. Stage 2 is described in more detail in the next Section.

In all the analyses, the constitutive model for the concrete in compression is based on plasticity theory. During the analysis of the effects of AAR on the concrete properties, for example, reductions in compressive strength and stiffness are included. A full description of the analysis is given elsewhere *(Cope et al 1994)*.

AAR EXPANSION ANALYSIS

This stage of the analysis simulates the period when AAR expansion is taking place. The total free expansion is applied in a number of increments. For each increment, the restrained expansions in the principal stress directions throughout the concrete, are determined at each sampling point from a family of curves of the same form as the stress-expansion relationship shown in Fig. 1. An initial strain analysis is then carried out using the restrained expansions as the initial strains. Generally, the member will not be in equilibrium. Therefore, the analysis is repeated using constitutive equations dependent on expansion level until convergence occurs. When convergence has occurred, the properties of concrete are modified, according to the values of expansion in principal strain directions throughout the concrete. The above procedure is then repeated until the total free expansion has been applied.

Since the material property changes are applied at the end of each incremental step, the incremental free expansion is chosen to be small, typically of the order of 0.02mm/m, in order to reduce the accumulated error and the chance of numerical disturbances caused by sudden large changes in material properties.

EXAMPLES

The finite element program has been used to analyse a large number of reinforced concrete members, full details of which are given elsewhere *(Cope et al 1994)*. Description of two of the examples analysed are given below.

Expansion analysis

In order to predict the strength and stiffness of a reinforced concrete member realistically, it is essential to initially predict the expansion and induced stresses within it to a reasonable degree of accuracy. It has been shown that the proposed expansion model is capable of predicting, relatively accurately, expansions of symmetrically reinforced specimens *(Cope et al 1994), (May et al 1992).* The inclusion of the expansion model into a finite element program, together with the models for cracking and change of material properties due to expansion, allows expansion analyses to be carried out, realistically, and on a wide range of reinforced concrete members. In this section, examples are given to verify the approach.

Example 1: Singly reinforced beam conditioned without load

Fig. 2 shows details of the smaller, singly reinforced beams tested by Cope *(Cope 1992).* These beams were prepared with a special mix designed by the Building Research Establishment and conditioned under water at 38°C to accelerate the AAR expansion. Expansions of the beams were monitored during the period of AAR at various positions and directions. The measurements taken included horizontal expansions along the top, horizontal expansions along the reinforcement and vertical expansions. The average values of the measured expansions were compared with the average free expansions of 100 x 200 mm cylinders, made of the same concrete and stored in the same conditions. These studies provided detailed information on the differential expansions on the surfaces of reinforced concrete members and the relationship between free and restrained expansions. The results of the tests have been used to verify the model for AAR expansion.

Fig. 3A shows the finite element representation of the beam. The data used for the analysis are given in Table 1. Concrete properties measured are without the effects of AAR. Figs. 3 B, C, and D show, respectively, the deformation of the beam, the cracks due to tensile stress in concrete caused by the deformation of the beam and the stresses induced in the reinforcement at a free expansion of 5 mm/m. The deformation shown in Fig. 3B has been amplified by a factor of 5.

Table 1 Parameters Used In Analyses

	Example 1	Example 2
Concrete Compressive strength (N/mm²)	50	45
Tensile strength (N/mm²)	2.5	2.5
Elastic modulus (kN/mm²)	40	40
Poisson's ratio	0.2	0.2
Ultimate crushing strain	0.0035	0.0035
Steel Yield stress (N/mm²)	400	560; stirrups 400
Elastic modulus (kN/mm²)	210	200

The average expansions measured on the beams are plotted against the average free expansions of the cylinders in Fig. 4. The predictions using the finite element analysis and those from elastic analysis assuming the expansions to be stress independent are also shown.

Fig. 5 shows the predicted stress distributions induced by AAR expansion through the mid-span section of the beam at different levels of free expansion ranging from 0.4 to 5 mm/m.

Fig.6 shows the predicted uniaxial compressive strength and initial Young's modulus

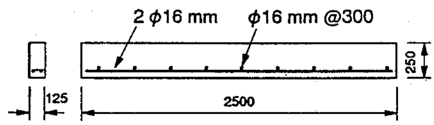

Fig. 2 Details of singly reinforced beam tested by Cope et al.

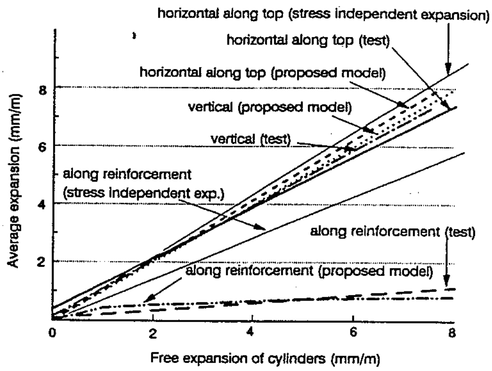

Fig.4 Comparison of expansion measurements with the predicted values using the proposed model and with those using elastic analysis assuming the expansion stress independent.

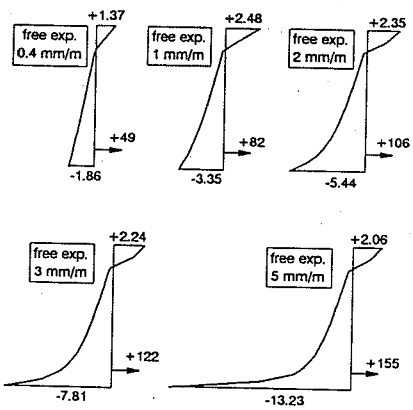

Fig 5. Predicted stress distributions across the sections of the singly reinforced beam tested by Cope et al, without loading during conditioning. Arrow represents reinforcement stress. Stress unit N/mm². Sign: + tension; - compression.

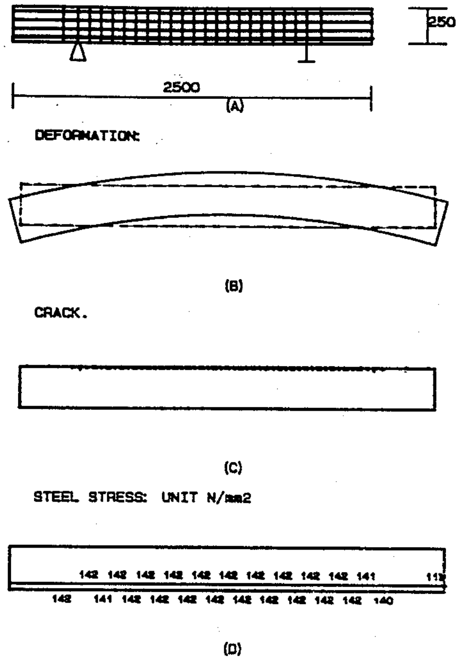

Fig. 3 A) Finite element model for the singly reinforced beam.
B) Deformation at free expansion of 5m/m (amplifying factor 5).
C) Cracking at top due to induced tensile stress in concrete.
D) Reinforcement stresses at free expansion of 5mm/m.

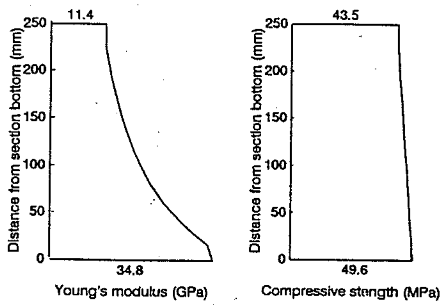

Fig. 6 Estimated Young's modulus and compressive strength through the section of singly reinforced beam conditioned unloaded.

across the mid span section when the free expansion is 5 mm/m using the material models described above.

From Figs 3 to 6 the following observations can be made:

a) The expansions due to AAR vary substantially throughout the depth of a beam. Fig.4 shows the difference between the expansion along the top and that along the reinforcement. The expansions predicted using elastic theory with stress-independent expansion vary less. The relatively lower expansion along the reinforcement demonstrates the sensitivity of AAR expansion to stress. The model proposed, taking into account stress history, was able to predict all the experimentally measured expansions closely. The differences between the predictions using the stress-independent expansion approach and the experimental results are significant. Strains predicted on the basis of stress-independent expansions are, therefore, not acceptable in the analysis of expansion effects due to AAR.

b) The distribution of stresses in a reinforced concrete member can be highly non-linear, Fig.5. This is a consequence of the shape of the stress-expansion curve. It can be seen that, in an un-symmetrically reinforced member, the induced compressive stress can be much higher than that which could be induced in uniformly restrained concrete. The maximum stress that could be induced in a uniformly restrained concrete is about 4 N/mm^2 (which would have prevented further expansion in the direction of the reinforcement) while the induced stress predicted at the soffit of the beam was about 13 N/mm^2.

c) The short vertical cracks along the top of the beam, shown in Fig.3C, are caused by the tensile stress generated by the differential expansion. AAR expansion can, therefore, induce tensile cracks in reinforced sections, in addition to those caused by the expansion of the gel within the aggregate and cement paste. In this example, tensile cracks along the top of the beam occurred at free expansions between 0.4 mm/m and 1 mm/m. After a free expansion of 2 mm/m, however, the cracks did not extend much further into the beam but widened slightly as the free expansion increased to 5 mm/m.

d) The predicted variations of the uniaxial compressive strength and the initial Young's modulus through the mid-span section for a free expansion of 5mm/m shown in Fig 6 are used in the analysis of the affected beam when it is subsequently subjected to external load, which is increased until member failure. It can be seen that the compressive strength is less sensitive to expansion than the Young's modulus: the maximum reduction in the strength, as determined from cube tests, was about 10% and the reduction in Young's modulus was up to 70%. This is in agreement with the trends in the material property-expansion relationship reported by other investigators and found from the authors' tests.

Example 2 : Danish beams

Bach et al *(Bach et al 1992)* have reported tests on beams without shear reinforcement (within the shear span) to investigate the effect of AAR on the shear capacity of members with very well anchored tensile reinforcement. These beams were also reinforced such that the shear failure occured at a load when the reinforcement was at only a low stress level. They found that AAR increased the shear capacity from 55 kN to 130 kN, despite the concrete strengths being reduced due to the AAR. Two of the beams in the tests have been analysed and the results are presented in this section. These are a control beam, A0, and a reactive beam, A12, with a free expansion of 5 mm/m.

Details of the beams are shown in Fig.7. The longitudinal reinforcement was cold-drawn steel with a nominal yield stress of 560 N/mm². The shear span to total depth ratio was 2.5. The beams were provided with long anchorage zones for the longitudinal reinforcement and also with stirrups outside the shear spans. The input data for the analyses are given in Table 1. The load deflection curves for the beams are given in Fig. 8.

Fig. 9A shows the finite element mesh used for the beams. Fig. 9B shows the predicted deformation of the beam, conditioned without load, subjected to AAR with a free expansion of 5 mm/m. The deformation has been amplified by a factor of 5. The predicted stresses induced in the concrete and reinforcement at a free expansion of 5 mm/m are shown in Figs.10 A to C. Comparison of the predicted crack pattern of the beams with and without AAR at a load near the failure load is shown in Fig.11.

a) The predicted failure mode of the control beam was a brittle shear mode with a major shear crack band in the shear span, Fig.11 B. This is in agreement with the mode of failure observed from the tests. The predicted failure load was, however, higher than the experimental value. The predicted failure load was 68.7 kN while the experimental value was 54 kN. There are several possible reasons for this. Some of the data required for the analysis were not available and were therefore estimated and there is usually considerable scatter of experimental results for the shear mode of failure.

b) In Fig.8, line OB represents the predicted hogging of the reactive beam, which was conditioned without loading. Line BC is the load-deflection curve from the analysis of load testing after the free expansion had reached to 5 mm/m. Curve OA is the predicted curve for the control beam. The corresponding curves from the tests are compared with the predicted curves in the figure. It can be seen that the predicted load-deflection curves are in general agreement with the test data.

c) The crack patterns observed in the tests for beam A5, which is similar to beam A12, suggested that the beam affected by AAR also failed in a shear mode but with the major diagonal crack forming further away from the support than that in the control beam A0. Between the lower end of the major diagonal crack and the support there are a series of short cracks which formed a short distance away from the soffit. The crack pattern seems to have been correctly reflected in the analysis, Fig. 11 A, although perfect bond was assumed.

d) The crack pattern described in paragraph c) above can be related to the stress distributions, shown in Fig.10, induced by AAR expansion. The compressive stresses induced in the concrete in the bottom of the member inhibit the tensile cracks occurring within a distance from the support, assuming that no bond slip occurs. Also there is vertical compression due to restraint of expansion due to the stirrups. Further away from the support, the tensile crack is delayed by the induced compressive stress. This delays the formation of the critical diagonal crack and forces the position of the crack away from the support. A series of short cracks in between the lower end of the critical diagonal crack and the supports observed in the tests for beam A5, were probably the results of the combination of the horizontal compressive stress and the

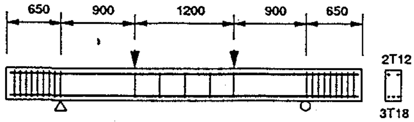

Fig. 7 Details of the beams tested by Bach et al.

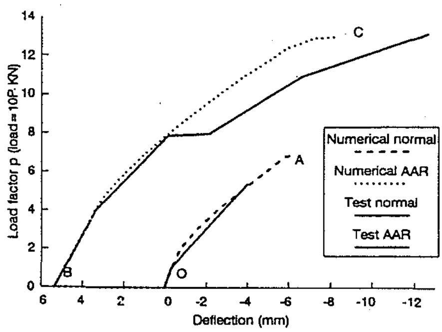

Fig. 8 Numerical and experimental load-deflection curves of the beams tested by Bach et al. The deflection measured on soffit under loading point.

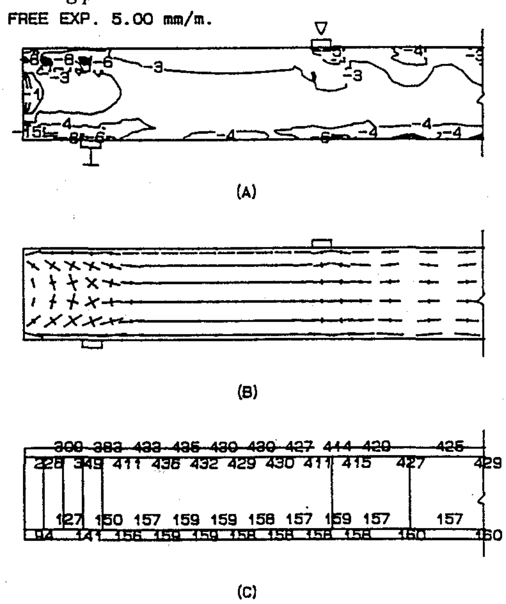

Fig. 10 (A) Concrete principal stress contours.
(B) concrete principal stress directions.
(C) reinforcement stresses in the reactive beam at free expansion of 5mm/m. Stirrups shown in (C) have yielded, stress - 400 N/mm².

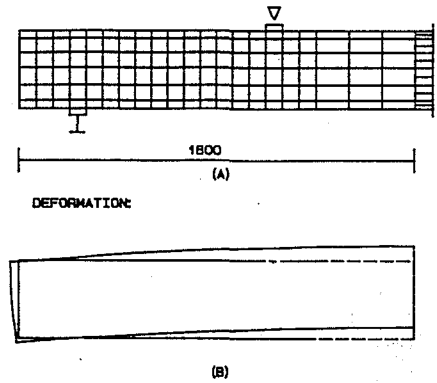

Fig. 9 (A) Finite element model for the beams tested by Bach et al.
(B) Deformation at free expansion 5mm/m

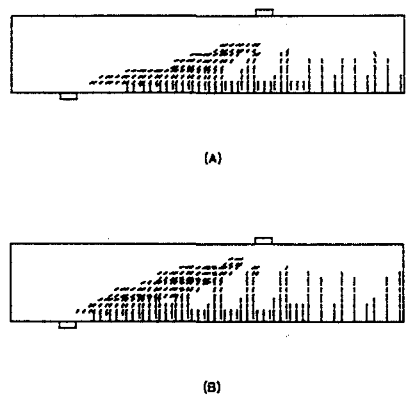

Fig. 11 Predicted crack patterns of the beams tested by Bach et al (A) with AAR and (B) without AAR.

shear stress. However, they may indicate local bond failure, which was not modelled in the analysis.

e) According to the analysis, when the control beam failed in shear, the maximum stress in the reinforcement was only 270 N/mm^2, which was less than half of the yield stress of the cold-drawn steel. For the reactive beam, the maximum predicted stress in the reinforcement was 540 N/mm^2, which was almost the yield stress, 560 N/mm^2, when the beam failed. The strength of the reinforcement was, therefore, fully exploited in the reactive beam. This was due to the prevention of the 'premature' shear failure, as in the unaffected beam, by the induced compressive stresses.

CONCLUSIONS

The agreement between the results from the analyses and the tests are generally good. The examples presented demonstrate and test the method. The technique provides useful insights into structural behaviour and explains test data. The tests on beams demonstrated that the effects of AAR on structural performance cannot be easily generalised. Its effects depend on the details of individual members and the loading regime during the expansion period. It has been demonstrated that the method described can provide a useful tool to investigate AAR effects in a range of structures.

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