FRACTAL ANALYSIS OF CRACKED SURFACE IN AAR CONCRETE

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ABSTRACT

The purpose of this study is to develop a method which can quantify aspect of damaged surface in concrete subjected to alkali aggregate reaction (AAR) in terms of fractal geometry. The paper presents a new evaluation method with fractal dimension analysis for AAR cracking. In the fractal analysis, cracking pattern of the damaged surface was digitized by using an image analysis which used box counting method. The results indicated the conditions of fractal analysis, the fractal character and the classification of AAR crack. Also, we investigated relationships between fractal dimension, AAR expansion and width of crack.

***Keywords*:** Fractal Analysis, Box Counting Method, Fractal Dimension, Lower Limit of Division, Number of Division, Expansion, Width of Crack, AAR

INTRODUCTION

Since Mandelbrot introduced a new geometry concerning Fractal[1], many disciplines have utilized the concepts of fractal. However, studies on the application of these concepts to cracked concrete have been scarce at best. A traverse method has been employed to evaluate cracked surface of AAR concrete for amount and width of the crack. So far, however, the concepts and method of quantifying the cracking aspect of the concrete have not been developed. This study is to introduce a new approach - fractal geometry concept - to the AAR crack. This concept provides useful tools for evaluating characterization of cracked surface in AAR concrete.

The concept of fractal has been successfully applied to a large number of phenomena, among them such as properties of mineral particles, crack of rock, cracking in damaged concrete, crack of pavement-concrete and landscape. Fractal is a shape made of parts similar to the whole in some way, and means irregular shaped figures characterized by a number D, called fractal dimension. D in Euclidean geometry is an integer and equals 1,2,3 for ordinary lines, surface and volumes, respectively. However, in the fractal situations, D can be a non-integer. The notion of a fractal between appropriate lower and upper dimensional bounds may be applied to comminution if the size distribution is fractal. In order to understand the relevance of fractal geometry for AAR crack it is necessary to discuss conditions of the fractal analysis, and investigate relationship between fractal dimension and AAR factors. In the study, we applied the concept of fractal to evaluating the cracks in damaged concrete due to AAR. Also, the simple traverse method was adopted to measure width of the cracks.

1. Fractal Dimension Analysis

The fractal dimension was analyzed to quantify the aspect of surface damaged concretes due to AAR. The objects of the analysis were cracking patterns of every description from the concretes, and box counting method was employed in the analysis. The fractal dimension depending on the box counting method is defined by cover theory as follows. The crack image is covered with square grids with size $r_0 \times r_0$, and changed the size of grids on given image. A simple paradigm for this box counting method can be constructed by referring to Fig.1. N indicates the number of

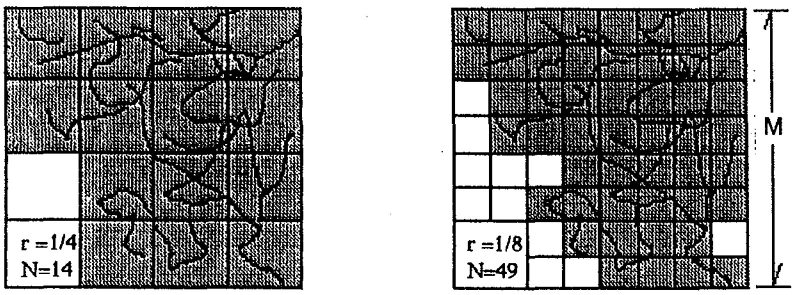

Fig. 1 Square cover method - grids of various size

square grids needed to cover the cracks (N changes with r_0). In the limit of r_0:

$$N \propto r_0^{-D} \qquad (1)$$

where D is fractal dimension depending on the particular characteristics of the phenomena. N is plotted as a function of division size r_0, observation is fitted by linear regression. The fractal dimension D may be determined by finding the slope of Log N(r) as a function of Log r, and is better to use the following equation[2)].

$$D(r) = -\frac{d \, \mathrm{Log} \, N(r)}{d \, \mathrm{Log} \, r} \qquad (2)$$

where $r=r_0/M$, M is a maximum length of crack.

The flow chart of the fractal analysis is given in Fig.2. In the fractal analysis the image is divided with changing size of grid from upper limit to lower limit. The number of square grid covering cracks in the processed image is counted by thinning the original binary image of crack. The grid containing at least one thinned crack is one picture element - divided square grid. Crack width is not considered in the fractal analysis as the box counting method.

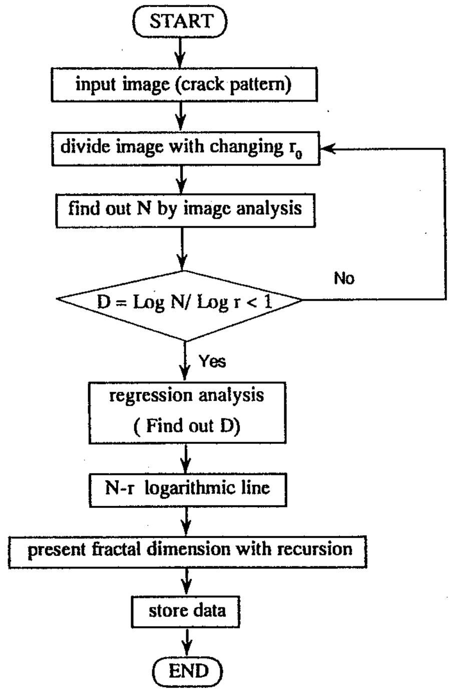

Fig. 2 System Flow of Fractal Dimension Analysis

In addition, ramiform aspect of cracking is quantified by shape factor (ϕ), and it is indicated as following expression:

$$M = \phi \sqrt{A} \quad (\phi \geqq 1) \quad (r_0 = M, \ N=1) \qquad (3)$$

where A is area of square covered crack.

Total length of crack to unit area is quantified by crack density (C.D) , it will be given by

$$\begin{aligned} C.D = L / A_0 &= \phi^{D} A^{D/2} \varepsilon^{1-D} / A_0 \\ &= M^{D} \varepsilon^{1-D} / A_0 \qquad (4) \end{aligned}$$

where L is total extension of crack ($L = N\varepsilon$), ε is lower limit of cracked length; A_0 is square area of fractal object.

Area of crack in term of crack ratio (C.R) is:

$$C.R = A/A_0 = (M/\phi)^2 / A_0 \qquad (5)$$

2. Result and discussion

Fig. 3 shows a plot of N(r) as a function of r ($r = r_0 / M$) depending on slope with logarithmic line of N and r_0 to determine the fractal dimension D. The values of fractal dimension range from 1 for lines to 2 for surface between appropriate lower and upper dimensional bounds. In the example the same division number was adopted.

2.1 Condition of fractal dimension analysis

In the fractal dimension analysis within box counting method, it was defined that the maximum division length (upper limit of fractal) corresponds with maximum extension of crack[3]. The lower limit of fractal is defined as follows. A simple example for this lower limit is constructed by referring to Fig.4. Because D in Euclidean geometry is an integer and equals 1 for ordinary lines, the size of division at which its fractal dimension is D=1 should be defined to lower limit of fractal. From this plot, that there is no longer fractal if its fractal dimension is D<1.

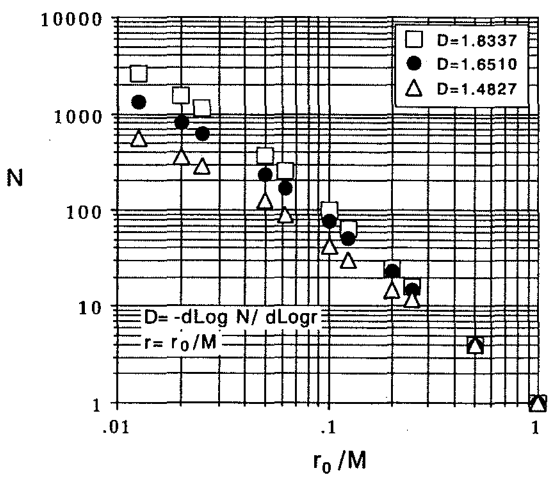

Fig. 3 An example analyzed fractal dimension (D)

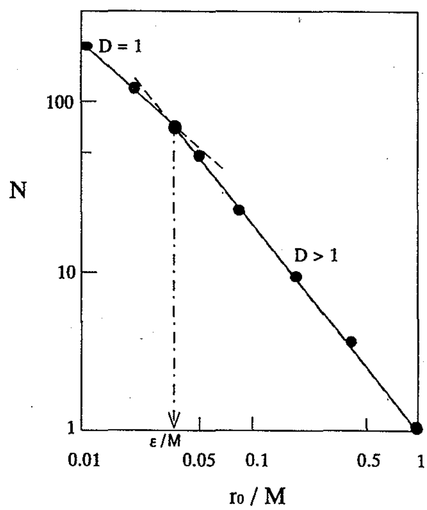

Fig. 4 Lower limit of fractal

The minimum division size (lower limit of fractal) for AAR crack and its effect on value of fractal dimension are investigate as follows.

The effects of lower limit of division (minimum division r_0) on result of analysis with slope for one upper limit are shown in Fig.5 and 6. This lower limit indicates a adopted minimum length to change division size by box counting method. The numeral in the explanatory notes indicated the division number. The fractal dimension increased with the increase in the minimum division size. These quantities of fractal dimensions were approximately equal when the same lower limit was employed. It is suggested that the division number did not influence the result of fractal dimension analysis in the same lower and upper bounds.

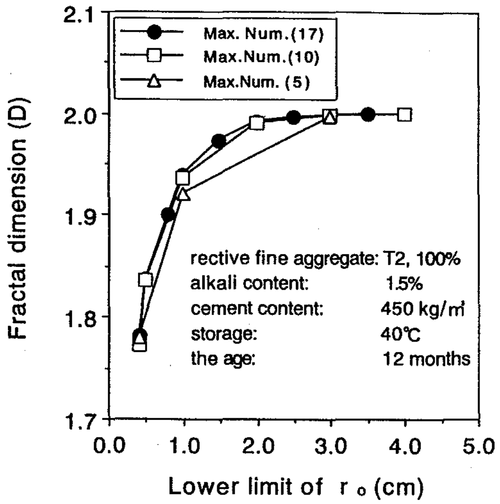

Fig. 5 Relationships between D and lower limit of r_o

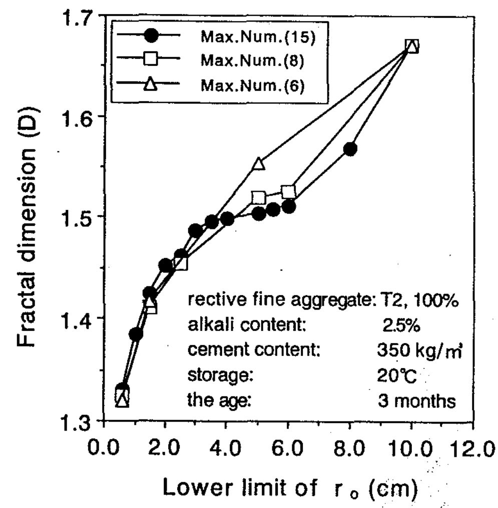

Fig. 6 Relationships between D and lower limit of r_o

The relationship between minimum division size and fractal character is shown in Fig.7. In this investigation the reactive fine aggregate with a maximum size of 5mm and the reactive coarse aggregate with a maximum size of 10mm were used in the AAR concretes, and the crack patterns of these concretes were analyzed with box counting method. From the results, there was a substantial division size range over which the fractal dimension is D>1, and when the division size (r_0) was smaller than maximum size of reactive aggregate, fractal dimension was D<1 in which there is no

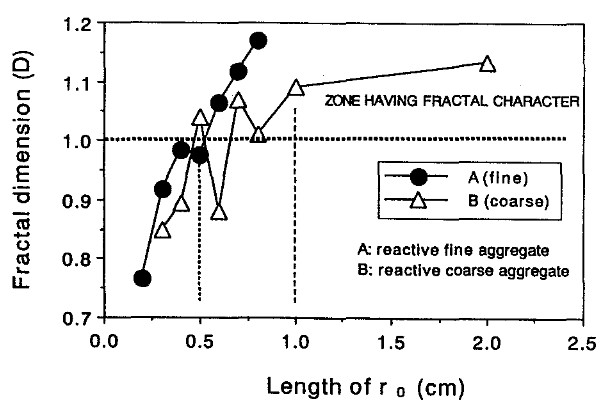

Fig. 7 Lower limit of r_0

fractal character. Therefore, it can be confirmed that the lower limit of division having fractal character should correspond the maximum size used reactive aggregate at least.

2.2 Fractal dimension and expansion

The results of fractal dimension analysis for AAR crack are presented in Table 1. The results in the table showed relationship between fractal character values and expansion. From this table, it is appeared that aspect of cracked surface in AAR concrete was classified to the three categories; linear, planar and alligator and were expressed by fractal dimension ranging from 1.0 to 1.2, 1.2 to 1.5 and over 1.5 respectively. When the aspect was alligator, crack density was larger than 0.1, and the expansion of concrete reached to over 1.0%.

Table 1. Analyzed Fractal Characteristic Values and Expansion

Fractal dimension (D)	Shape factor (ϕ)	Crack density (C.D)	Crack ratio (C.R) (%)	Expansion (%)
1.80~1.89	1.50~1.35	0.74~0.87	44~56	0.40~0.80
1.70~1.79	1.81~1.49	0.35~0.79	31~49	0.20~0.56
1.60~1.69	2.13~1.67	0.27~0.55	22~37	0.17~0.36
1.50~1.59	2.40~1.86	0.17~0.37	12~29	0.10~0.23
1.40~1.49	3.26~2.76	0.15~0.23	9 ~18	0.05~0.14
1.30~1.39	3.80~2.45	0.11~0.16	6 ~16	0.05~0.12
1.20~1.29	4.62~3.30	0.05~0.11	5 ~11	0.05~0.07
1.10~1.19	4.91~3.74	0.04~0.07	4 ~7	0.05~0.06

Fig.8 is a plot of the relationship between fractal dimension and expansion. The fractal dimension increased with the increase in the expansion, and this increase rate was inclined to gradually less with the increase of expansion. The fractal dimension reached over 1.5 when the expansion was larger than 0.1% which is harmful for concrete structure[4]. The relationship between fractal dimension (D) and expansion (E) was revolved as following equation.

$$\text{Log } E = 1.72\, D - 3.48 \qquad (6)$$

The propagation of AAR crack is essentially due to expansion. Fig. 9 shows the relationship between the expansion and the total width of cracks between the measuring plugs (mm). The plot compressed that the total width of cracks was smaller than the expansion, and not necessarily the expansion. This result agreed with that from S.Diamond[5].

Fig.10 shows the fractal dimensions versus time development when two kinds of alkali were used. The fractal dimension increased monotonically with the increase in the ages, and was different with kinds of alkali. That is to say, the aspect of cracked surface and the amount of crack were different with the kinds even though the same alkali content was used. The fractal dimension increased slightly with the age after 6 months, and it can be considered that the change of crack in image was mainly from the increase in the its width.

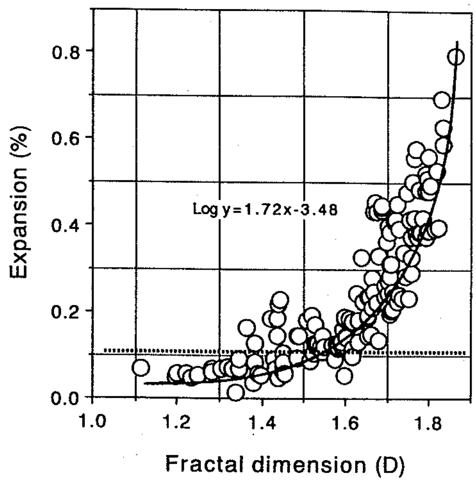

Fig. 8 Relationships between fractal dimension and expansion

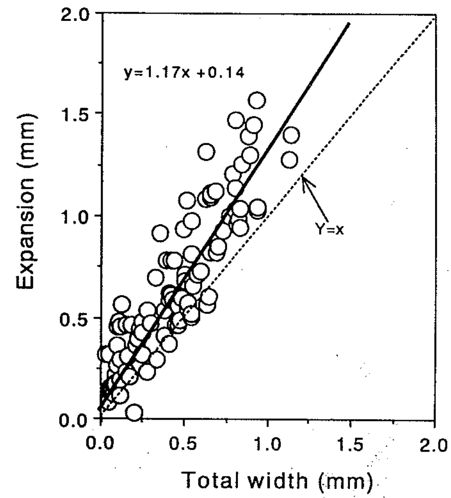

Fig. 9 Expansion and total width of cracks between the plugs

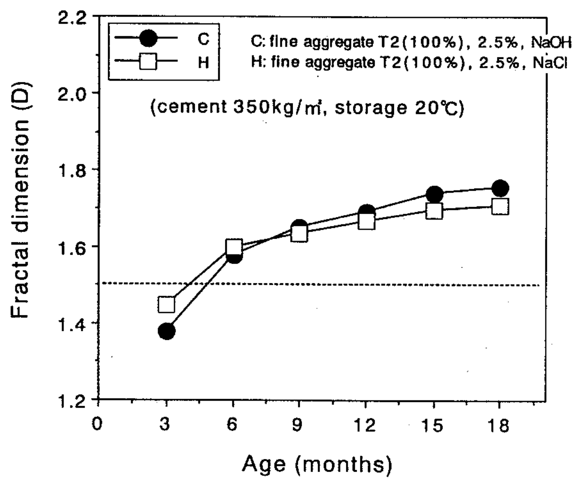

Fig. 10 Fractal dimension VS. time in 20℃, 100% RH. storage

Fig. 11 presents the crack patterns which were indicated by fractal dimension at the several ages. These crack patterns expressed a process in which the crack changed from linear to alligator, and were quantified by fractal dimension. It is observed that the effectiveness of this estimate with fractal dimension analysis was confirmed.

3. Conclusion

The box counting method with fractal dimension analysis for AAR cracking characterization was employed in this study. Through this method, the aspect of cracked surface of AAR concrete was analyzed and evaluated. The following was clarified by this study.

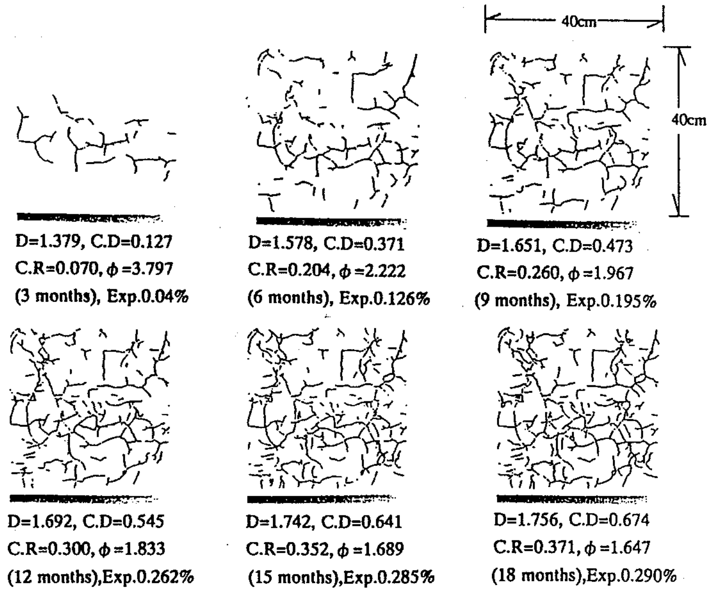

Fig. 11 Crack patterns at the several ages

(1) Cracking aspect of AAR concrete has fractal character and can be quantifically characterized by fractal values.

(2) The fractal dimension increased with the increase in the adopted minimum division size. The division number did not influence the results of fractal dimension analysis.

(3) The lower limit of division having fractal character should correspond with the maximum size of reactive aggregate used.

(4) AAR cracking aspect was classified to linear, planar and alligator by fractal dimension ranging from 1.0 to 1.2, 1.2 to 1.5 and over 1.5, respectively. It was observed that the expansion of concrete reached to over 1.0%, and the fractal dimension was D>1.5 at which the aspect of crack was alligator.

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