# THE MODELLING OF THE EFFECTS OF AAR EXPANSION ON REINFORCED CONCRETE MEMBERS

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It has been shown that AAR induced expansion can be reduced significantly by the application of compressivestress. The effect of this reduction in expansion has not been previously included in the analysis of AAR specimens. This paper describes an incremental analysis for symmetric sections which takes account of this phenonemon and compares the theoretical results with those obtained in tests. An incremental numerical analysis based on similar assumptions is then described that can be used for general sections. The results from this analysis are also compared with experimental data.

## INTRODUCTION

Early attempts to analyse the effects of AAR expansion on reinforced concrete structures utilised free expansions obtained from unloaded control specimens. The attempts at estimating the expansions in reinforced specimens with the full value of Young's Modulus led to predictions of strains often orders of magnitude larger than those measured. Recent work, particularly that of Hobbs (1), (2), Clayton (3), and Chana (4), have shown that expansions of reactive specimens can be dramatically reduced even under a comparatively small compressive stress.

The inclusion in a specimen of reinforcement will obviously lead to a variable restraint stress as expansion proceeds. In order that that assessment of AAR affected structures can be carried out a technique is required to determine the distribution of strains and stresses in the section during the AAR.

The paper demonstrates the effects of the influence of stress on expansion through a closed form solution for a symmetric section. After showing that good agreement with experimental results can be obtained a brief description is given of the extension of the method to non-symmetric sections with non-linear stress-expansion curves. For these analyses an iterative numerical approach using finite elements is described. An example is given which demonstrates the effectiveness of the method. Further analyses have been carried out and these are described elsewhere (6).

### Effect of Applied Stress

Application of a constant applied compressive stress restricts the expansion that occurs during AAR. The empirical relationship between the applied stress, $\sigma_c$ and the expansion $\varepsilon_{ex}$ assumed for the analysis is shown in

Fig.1, which also shows the experimentally obtained relationship, and is given by

$$\varepsilon_{ex} = 0; \quad \text{for} \quad \sigma_c < \sigma_c^o \quad \ldots\ldots (1a)$$

$$\varepsilon_{ex} = \left(1 - \frac{\sigma_c}{\sigma_c^o}\right) \varepsilon_{ex}^{fo}; \quad \text{for} \quad \sigma_c^o < \sigma_c < o \quad \ldots\ldots (1b)$$

$$\varepsilon_{cx} = \varepsilon_{ex}^{fo}; \quad \text{for} \quad o < \sigma_c \quad \ldots\ldots (1c)$$

where $\sigma_c^o$ is the stress at which zero expansion occurs and $\varepsilon_{ex}^{fo}$ is the free expansion of a specimen with no applied stress. It is possible to use other non-linear relationships for the expansion-applied stress curve and this is discussed later.

Consider the symmetric section shown in Fig.2. The member is of original length L and at some time has undergone an extension ΔL such that the strain $\Delta L/_L$ is ε. The section now undergoes a further small increment extension dL leading to a strain dε which can be considered as an expansion, due to AAR, of the unrestrained concrete $dL_e$ with a corresponding strain $d\varepsilon_{ex}$, and the subsequent reduction in expansion $dL_{ce}$, corresponding strain $d\varepsilon_{ce}$, due to the restraining effect of the reinforcement.

Equilibrium requires that

$$E_s A_s d\varepsilon + E_c A_c d\varepsilon_{ce} = 0$$

letting $E_s/E_c = m$ and $A_s/A_c = \rho$ where $E_s$ and $E_c$ are Young's moduli of steel and concrete respectively and $A_s$ and $A_c$ are the areas of concrete respectively then

$$d\varepsilon_{ce} = -m\rho d\varepsilon \quad \ldots\ldots (2)$$

Compatibility requires that

$$d\varepsilon = d\varepsilon_{ex} + d\varepsilon_{ce} \quad \ldots\ldots (3)$$

If the stress induced in the concrete is within the range $\sigma_c^o$ to o then the incremental expansion due to AAR in an unrestrained specimen would be given by

$$d\varepsilon_{ex} = \left(1 - \frac{\sigma_c}{\sigma_c^o}\right) d\varepsilon_{ex}^{f} \quad \ldots\ldots (4)$$

The stress in the concrete is given by

$$\sigma_c = -E_s\rho\varepsilon \qquad (5)$$

thus

$$d\varepsilon = (1+\frac{E_s\rho}{\sigma_c^o}\varepsilon)d\varepsilon_{ex}^f - m\rho d\varepsilon \qquad (6)$$

and

$$\frac{d\varepsilon}{d\varepsilon_{ex}^f} = \frac{1}{1+m\rho}(1+\frac{E_s\rho}{\sigma_c^o}\varepsilon) \qquad (7)$$

from which

$$\varepsilon = -\frac{\sigma_c^o}{E_s\rho} + C\exp(\frac{E_s\rho}{\sigma_c^o(1+m\rho)}\varepsilon_{ex}^f) \qquad (8)$$

The constant c, which is determined using the condition that when $\varepsilon_{ex}^f = 0$, $\varepsilon = 0$, is given by

$$C = \frac{\sigma_c^o}{E_s\rho} \qquad (9)$$

Hence

$$\varepsilon = \frac{\sigma_c^o}{E_s\rho}(\exp(\frac{E_s\rho}{\sigma_c^o(1+m\rho)}\varepsilon_{ex}^f)-1) \qquad (10)$$

This equation gives the relationship between the final strain and the free expansion for a symmetric section provided the reinforcement remains elastic, ie $\varepsilon<\varepsilon_y$. If $\varepsilon$ is greater than $\varepsilon_y$, the yield strain of the steel, then yield of the reinforcement occurs. For any further increase in expansion of the specimen the stress within the concrete remains constant. If the free expansion at yield of the reinforcement is $\varepsilon_{ex}^{fy}$ then the additional expansion of the specimen after yield has occurred is given by equation (1b) and is

$$(1-\frac{\sigma_{fy}}{\sigma_c^o})(\varepsilon_{ex}^{fo} - \varepsilon_{ex}^{fy}) \qquad (11)$$

where $\sigma_{fy}$ is the stress in the concrete following yield. The final expansion is

$$\varepsilon = \varepsilon_y + (1 - \frac{\sigma_{fy}}{\sigma_c^o})(\varepsilon_{ex}^{fo} - \varepsilon_{ex}^{fy}) \qquad (12)$$

The free expansion when yield of reinforcement occurs, $\varepsilon_{ex}^{fy}$, can be obtained from Equation 10 by setting $\varepsilon = \varepsilon_y$ and solving thus

$$\varepsilon_{ex}^{fy} = \frac{\sigma_c^o(1+m\rho)}{\rho E_s} \ Ln(1+ \frac{\rho E_s \varepsilon_y}{\sigma_c^o}) \quad \ldots\ldots (13)$$

The steps in the analysis of a symmetrically reinforced section are the expansion of the specimen is calculated using Equation 10. If $\varepsilon > \varepsilon_y$ then $\varepsilon_{ex}^{fy}$ is calculated from Equation 13 and the final expansion is determined from Equation 12.

## Examples

In order to check the validity of the proposed analysis estimates using Equations 10, 12 and 13 have been made of the total expansion for a number of members tested by others.

## Michihiko's Tests (5)

The members were 2.0m long and 200mmx200mm square with 4 longitudinal bars placed at the corners and stirrups. The assumed properties and experimental and theoretical results are given in Fig.3. The theoretical analyses have been carried out at the three free expansion levels which correspond to the values measured on the control specimens. It can be seen that the agreement is good. Also shown on the figure is the overall expansion - steel ratio relationship for the members assuming that the expansion is independent of applied stress.

## Hobbs Series I Tests (2)

The members were symmetrically reinforced specimens. The assumed properties and experimental and theoretical results are given in Fig.4. As with the comparisons of Michihiko's Tests the agreement is good.

## Inclusion of general relationships for stress-expansion curves

The analysis described is capable of dealing only with linear changes in the stress-expansion curve, Equation 1b. In order to deal with non-linear forms of the stress-expansion curve, to closer match the experimental curve indicated in Fig.1 then, even for symmetric sections, a numerical incremental approach has to be adopted. For an initially stress and strain free specimen this involves applying a small increment of free expansion. Since in this case the stress is zero the initial incremental expansion due to AAR, $\delta\varepsilon_{ex}$, in an unrestrained specimen will be the free expansion. The incremental expansion of the specimen is given by

$$\delta\varepsilon = \frac{\delta\varepsilon_{ex}}{1+m\rho} \quad \ldots\ldots (14)$$

Hence the stress at the end of the increment and the total strain can be calculated. Further increments of free expansion are then applied, the incremental expansion due to AAR being determined using the non-linear expansion-stress model assumed with the stress being taken as that at the end of the previous increment. The incremental and total strains are then calculated. The process is repeated until the full free expansion has been applied. Account of yielding of reinforcement can be taken. Further details are given in reference (6).

## Extension of analysis to non-symmetric sections

In order to extend the analysis to non-symmetric sections with general applied stress-expansion curves the model for applied stress-expansion has been included in an elastic finite element program developed by Naji (7). The program which builds on the procedure described in the previous section is iterative and is described in more detail elsewhere (6). The program has been used to analyse beams tested by Cope and Slade (8). Details of the beam are given in Fig.5. A cubic approximation of the applied stress-expansion curve was used with a limiting stress of $4N/mm^2$. The calculated expansions together with those measured at various positions are given in Fig.6, good agreement can be seen to exist. The calculated stress and strain distributions at the midspan section are given in Fig.7.

Both analysis and the test indicate strains close to the free expansion level at the top of the member, Fig.7. The stress plots, assuming elastic properties to the concrete show the highly non-linear variation of stress due to the assumed relationship between stress and expansion.

## Conclusions

The analysis proposed has been shown to be able to predict, in general terms, the effect of expansion due to AAR on reinforced concrete members. It has been shown that the inclusion of the reduction of expansion due to the application of compressive stress is necessary to obtain realistic estimates of the expansions.

The analysis has been used on a wide range of specimens, some results are given in Cope et al (9), and it appears to give good predictions of behaviour of AAR affected members.

## Acknowledgements

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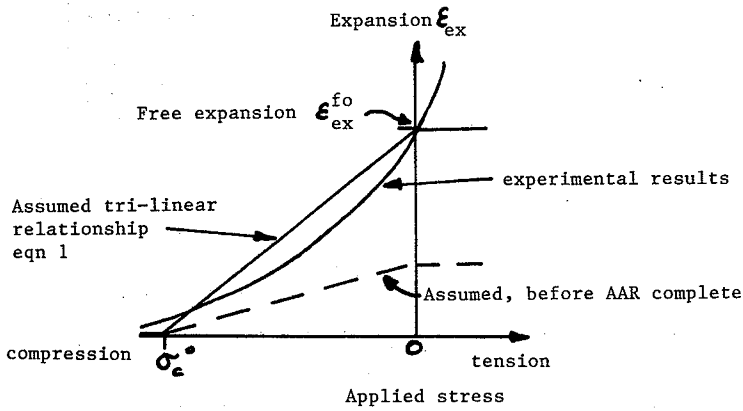



**Fig 1  Expansion-applied stress relationship**

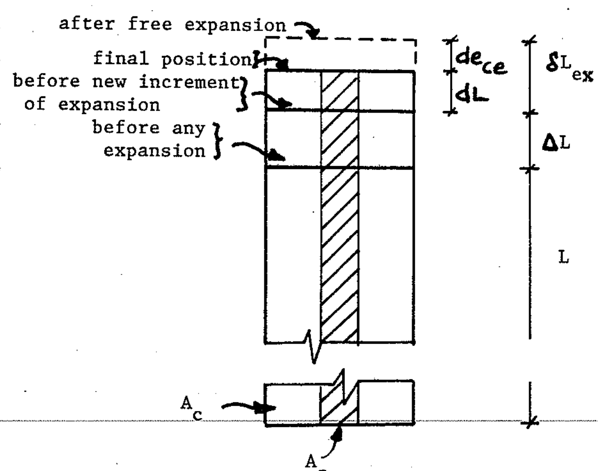



**Fig 2  Expansion of restrained specimen**

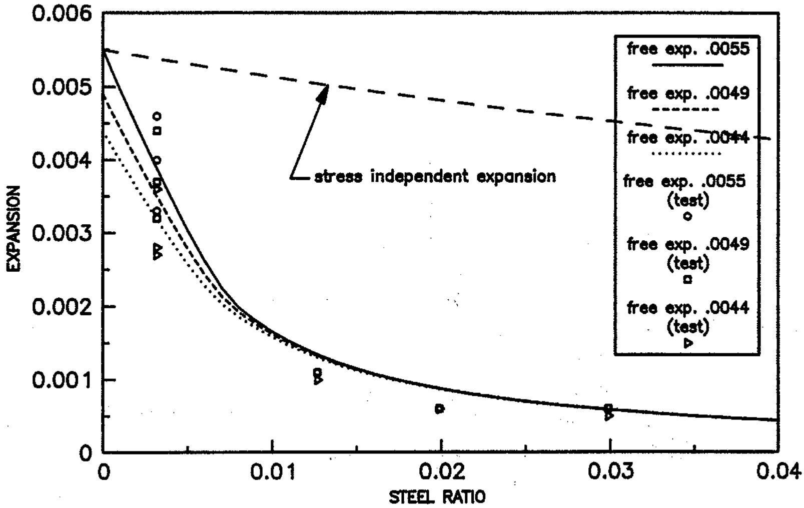



Fig.3 Analytical steel ratio–expansion curves and test results of Michihiko's beams.
(Ec=28000, Es=200000, Yield stress=400 MPa)

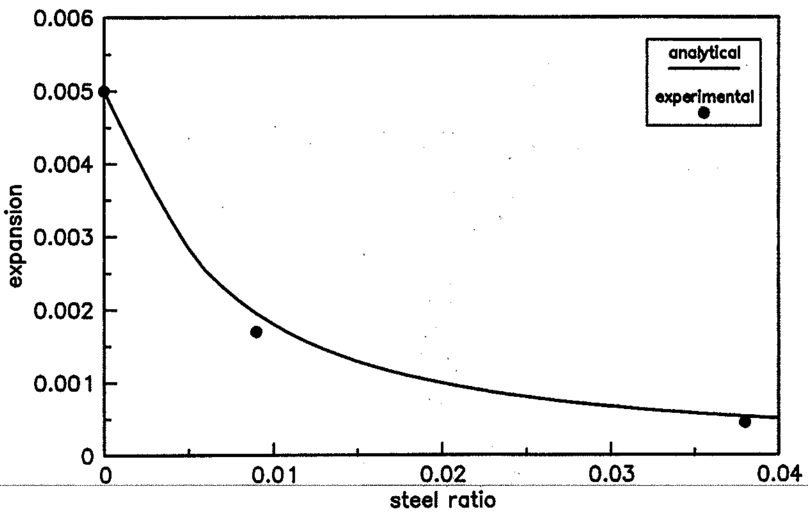
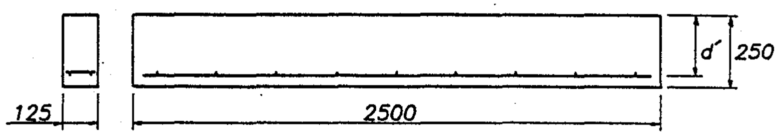



Fig.4 Analytical and experimental results of Hobbs test series I.
(Ec=24000,Es=200000,Yield stress=500 MPa)




*Concrete:* *C50 BRE Bridge Mix*
*Tension Steel:* *2 @ 16mm Ø Mild Steel bars*
*Transverse Steel:* *9 @ 16mm Ø @ 300 c/c*

**Fig 5 Details of Cope and Slade's (8) beam**

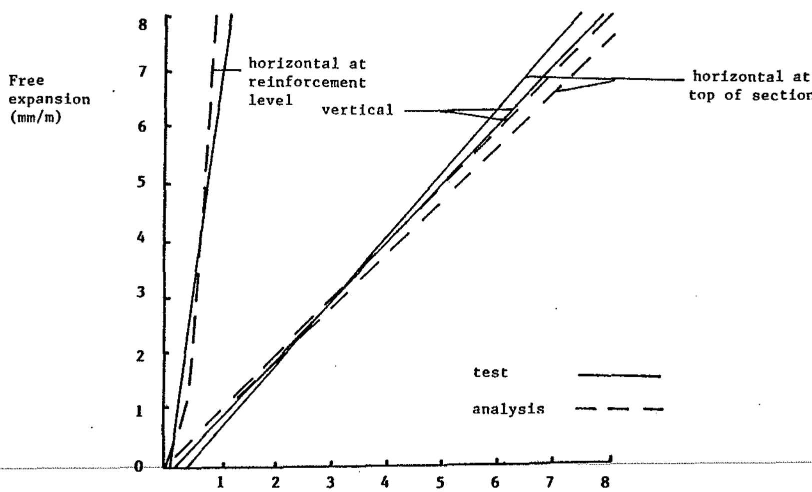



**Fig 6 Experimental and analytical results for Cope & Slade's (8) beam**

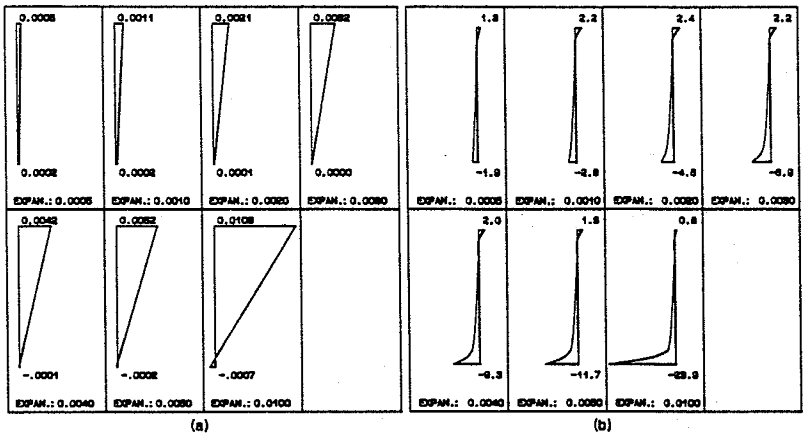


Fig 7 (a) Predicted strain distribution and

(b) estimated stress pattern at the midspan section of Cope and Slade beam.