ANALYTICAL EVALUATION OF EFFECT ON ASR OF RC STRUCTURE USING MESO-SCALE CONCRETE ELEMENT BY FEM

Maki Mizuta[1,*], Takayuki Kojima[2], Kazuhiro Kuzume[3]

[1]Ritsumeikan University, Dept. of Civil Engineering,
525-8577 SHIGA, Japan

[2]Ritsumeikan University, Dept. of Archtecture and Urban Design,
525-8577 SHIGA, Japan

[3]Kokusai Structural Engineering Corporation, 550-0005 OSAKA, Japan

Abstract

This study was aimed to propose a novel meso-scale concrete element in two-dimensional FEM analysis and to evaluate a cracking pattern and a confinement effect of stirrups on volumetric expansion associated with alkali silica reaction (ASR). The proposed concrete element consists of aggregate, mortar matrix and potential crack elements. First of all, a fundamental characteristic of the proposed concrete element was investigated by the analysis on a plain concrete. Modulus of elasticity of ASR ring element around an aggregate varied and its effect of swelling on cracking was focused herein. The next step was to investigate the confinement effect of stirrups on a concrete reinforced by stirrups. The differences in cracking pattern, amount of swelling between a plain concrete and a reinforced concrete and amount of swelling between the reinforced concretes with various quantities of stirrups were analytically evaluated.

Keywords: ASR, FEM, discrete crack element, cracking pattern, confinement effect of stirrup

1 INTRODUCTION

Since the concrete structures deteriorated by ASR were observed, the experimental studies on deterioration mechanism of ASR have been mainly reported. These previous researches have greatly contributed to the present maintenance works for ASR-damaged concrete structures. Unfortunately, the concrete structures severely deteriorated beyond expectation have been found still now on and the further damaged structures after rehabilitation have been also reported.

The swelling process and cracking pattern of concrete caused by ASR are affected by various factors. For example, (a) type of reactive aggregate, (b) confinement effect of reinforcement, (c) surrounding condition and (d) combination with other deterioration factors and so on can be listed. These factors may be an obstacle to the further development of research on ASR and of effective maintenance work. Therefore, not only the accumulation of the data measured by regular inspections but the analytical evaluation of a concrete deteriorated by ASR should be required.

Analytical study is an objective tool to understand a real phenomenon. Especially, the FEM analysis using a previously proposed discrete crack element [1, 2] can simulate the crack initiation and propagation in a concrete. Meso-scale concrete element with aggregate, mortar matrix and potential crack elements is newly proposed in this study. The analysis by the use of the element can represent volumetric expansion associated with ASR and radial cracking in mortar matrix caused by swelling of alkali silica gel.

This paper summarises the modelling of meso-scale concrete element by two-dimensional FEM analysis and the analytical results using this novel concrete element. The analysis of a concrete with/without stirrups can make clear the characteristics of alkali silica gel around an aggregate and the effect of stirrups on ASR expansion.

2 MODELING OF ELEMETS

2.1 Discrete crack element

Idealization of the discrete crack element in two-dimensional field is shown in Figure 1. The discrete crack element is composed of two isoparametric linear elements with three nodes and occupied the same position as nodes of the adjacent element. The discrete crack elements are inserted

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[*] Correspondence to: m-mizuta@se.ritsumei.ac.jp

between all the aggregate and mortar matrix elements as a potential crack element in advance. The x- and y-coordinates of node (1) and (4), (2) and (5), and (3) and (6) occupy the same position respectively before the occurrence of a relative displacement between two linear elements. Also the discrete crack elements have no volume.

The details of the discrete crack element along one-dimensional coordinate, s are given in Figure 1. The element is expressed in t-n coordinate, where t is the tangent of s coordinate and n is perpendicular to it. The relationship between transferring stress and relative displacement is as follows;

$$\{\sigma\} = \begin{Bmatrix} \sigma_n \\ \tau_t \end{Bmatrix} = \begin{bmatrix} C_{nn} & C_{nt} \\ C_{tn} & C_{tt} \end{bmatrix} \begin{Bmatrix} \delta_n \\ \delta_t \end{Bmatrix} = [C]\{\delta\} \tag{1}$$

where the matrix [C] is the property of the discrete crack element.

When the element is expressed in local coordinate, the stiffness matrix $[K_c]$ of the discrete crack element can be obtained by the numerical integration as following Equation 2.

$$\begin{aligned} [K_c] &= \int_s [B]^T [C][B] \cdot t \cdot ds \\ &= \int_{-1}^{1} [B]^T [C][B] \cdot t \cdot \sqrt{(dx/d\xi)^2 + (dy/d\xi)^2} \cdot d\xi \end{aligned} \tag{2}$$

where [B] is matrix between displacement and relative displacement, t is thickness of aggregate or mortar matrix element and is equal to 10 mm. Detailed information of the proposed discrete crack element can be found in References [1, 2].

The transferring stress, σ_n and crack width, δ_n in the discrete crack element was assumed to be related as shown in Figure 2. This relationship is referred by tension softening curve of concrete. The σ_n - δ_t relationship is not so important in this analysis and a constant value of $C_{nt}=C_{tn}=0$ N/mm^3 was assumed. And C_{tt} was also chosen to be a constant value, G/t, where G is shear modulus of rigidity of concrete (show Table 1).

2.2 ASR element

The ASR element was proposed to analyze a swelling process around an aggregate induced by ASR and to simulate the crack propagation in the mortar matrices after expansion. The finite element mesh of the ASR element is shown in Figure 3. This element consists of aggregate and mortar matrix elements, and two kinds of discrete crack elements. One is the ASR ring element between mortar matrix and aggregate elements and can represent a swelling process of alkali silica gel around an aggregate. The other one is the potential crack element. Also two types of the potential crack elements, the mortar crack element between mortar matrix elements and the aggregate crack element between aggregate elements are distinguished. All discrete crack elements were inserted at the beginning of the analysis.

Figure 3 also involves the stirrup elements and the boundary condition. Linear elements with two nodes were used for stirrups.

3 OUTLINE OF ANALYSIS

3.1 Flow

In ASR ring element, the non-linear relationship between amount of swelling, δ_{rn} and characteristic time, t is dominating as shown in Figure 4. The δ_{rn} - t relationship is as following Equation 3;

$$\delta_{rn} = \frac{1}{2700}\left(-2T^3 + 15T^2 - 24T + 1\right) \tag{3}$$

where T is equal to $\log_{10}t$.

In this analysis, the concrete is assumed to be a linear elastic body and the discrete crack elements between the mortar matrix elements have the non-linearity. Mechanical properties of mortar matrix and aggregate elements are listed in Table 2. The modulus of elasticity of aggregate crack element is given to be 20.0×E_{c0} (E_{c0} is equal to 3.0×10^4 N/mm^2). Table 3 summarizes the modulus of elasticity of each discrete crack element.

The flow of the analysis is as follows;

1) The amount of swelling is applied to the nodes of the ASR ring elements (show Figure 4).
2) C_{nn} in the discrete crack elements between the mortar matrix elements is modified. The iteration is carried out as the relationship between transferring stress and crack width is adjusted to the tension softening curve shown in Figure 2.
3) After the convergence is confirmed, the steps from 1) to 2) are repeated.
4) The analysis was finished after t is equal to 2000 days.

3.2 Factors

This analysis is accomplished in order to obtain the fundamental characteristic of the proposed meso-scale concrete element and to evaluate the mitigation effect of stirrups against ASR expansion. The analytical factors are listed in Table 4. And Table 5 summarizes the mechanical properties of stirrups.

4 RESULTS

4.1 Effect of modulus of elasticity of ASR ring element

The ASR ring element was defined to represent the expansion of alkali silica gel around the aggregates in this study. The characteristic of alkali silica gel, however, has not been clear yet. Hence three values of the modulus of elasticity of ASR ring element, E_r listed in Table 4 were chosen.

The relationships between amount of swelling and characteristic time and between crack width and characteristic time are shown in Figure 5 and 6, respectively. Because the similar tendency of the relationships in both cases of E_r=0.5×E_{c0} and 1.0×E_{c0} was observed, Figure 5 and 6 does not include the results of E_r=0.5×E_{c0}. And Figure 7 shows the cracking patterns in all cases of E_r.

Figure 8 shows the relationship between amount of swelling and E_r after 2000 days. Various E_r values are chosen to be from 0.1×E_{c0} to 20.0×E_{c0}.

Figure 3 also shows the points where the amount of swelling is measured. The amounts of swelling in x- and y-directions are the average of A and B, and C and D, respectively. Also, the average of A, B, C and D is the amount of swelling of the whole ASR element.

4.2 Effect of cross sectional area of stirrup

In cases I, III and IV, the relationships between amount of swelling and characteristic time and the cracking patterns after 2000 days are shown in Figure 9 and 10, respectively. The modulus of elasticity of ASR ring element is chosen to be 1.0×E_{c0} herein. These results reflect the mitigation effect of stirrup itself and of the cross sectional area of stirrup on ASR expansion.

The amounts of swelling in x- and y-directions are the average of A and B, and C and D, respectively (show Figure 3).

4.3 Effect of arrangement of stirrup

In case I and II, the relationships between amount of swelling and characteristic time and the cracking patterns after 2000 days are shown in Figure 11 and 12, respectively. The modulus of elasticity of ASR ring element is chosen to be 1.0×E_{c0} herein. The results of ASR element arranged stirrups in only x-direction are compared with them in both x- and y-directions.

The amounts of swelling in x- and y-directions are the average of A and B, and C and D, respectively (show Figure 3).

5 CONCLUSIONS

5.1 Fundamental characteristics of ASR element

In all cases, the amount of swelling in x-direction was approximately equal to that in y-direction and it reached to 0.016 mm after 2000 days (see Figure 5 and 6). Because of the symmetry of the proposed ASR element and of the boundary condition, the expected results of expansion were obtained. Also, the two kinds of modulus of elasticity of ASR ring element, E_r=1.0×E_{c0} and 10.0×E_{c0} did not much affect the relationship between amount of swelling and characteristic time.

On the other hand, the order of crack propagation was affected by E_r value (see Figure 7). The more the modulus of elasticity of ASR ring element, E_r increased, the earlier the crack between mortar matrix elements happened. When all mortar crack elements opened in case of E_r=0.5, 1.0 and 10.0×

E_{c0}, 300, 200 and 80 days passed, respectively. Also, the first crack occurred at the mortar crack element (6) and the last happened at (7) in all cases.

In case of $E_r=1.0\times E_{c0}$, the crack widths of all mortar crack elements were of even value, 0.007 mm after 2000 days. However, the crack widths in case of $E_r=10.0\times E_{c0}$ varied from 0.007 to 0.01 mm, and the crack width was maximum at the mortar crack element (7) and the minimum width occurred at the element (4).

Analytical results of various E_r values from 0.1 to $20.0\times E_{c0}$ were investigated as shown in Figure 8. Although the amounts of swelling varied from $E_r=0.3$ to $1.0\times E_{c0}$ were stable, the other results of amount of swelling were of uneven and were less than them. Especially, the amounts of swelling in cases of E_r less than $0.3\times E_{c0}$ tended to decrease and only a few mortar cracks happened.

These analyses indicated that the modulus of elasticity of ASR ring element had a large effect on the cracking pattern and the amount of swelling. The modulus of elasticity less than that of normal concrete obviously affected the analytical results. Therefore E_r value at further analyses of ASR element with stirrups was fixed to be $1.0\times E_{c0}$.

5.2 Mitigation effect of stirrups against ASR expansion

Results of the ASR element with/without stirrups and of varied cross sectional areas of stirrups were analytically investigated (show Figure 9 and 10). In case IV without stirrups, the ASR element uniformly expanded. The amounts of swelling in x- and y-directions were approximately twice of the given displacement of ASR ring element. Also, all radial mortar cracks happened after 2000 days and the crack widths of them were approximately 0.007 mm.

On the other hand, the amounts of swelling in x-direction of case I and III were much less than these of case IV and the number of the mortar crack was decreasing. Stirrups also contributed to the decrease in volumetric expansion in y-direction after 2000 days. The increase in cross sectional area of stirrup did not act against the ASR expansion very much and it led to the slight decrease in amount of swelling in x-direction. The maximum crack width at the element (6) was more than that without stirrups and it was 0.03 mm. Hence the mitigation effect of stirrups against ASR expansion could be analytically evaluated.

Effect of arrangement of stirrups was analytically investigated as shown in Figure 11 and 12. In case II with arrangement of stirrups in x- and y-directions, both the amount of swelling and the number of cracks were further decreasing than the ASR element reinforced by stirrups in only x-direction in case I. The maximum crack width at the mortar crack element (6) in case II was 0.027 mm. Therefore it suggested that the arrangement of stirrups in plural directions might be more effective on the mitigation of ASR expansion than that in singular direction.

6 CONCLUSIONS

(1) Meso-scale concrete element with potential crack elements by two-dimensional FEM was newly proposed. Expansion of alkali silica gel and radial crack propagation around an aggregate could be simulated by the use of the proposed element.

(2) It was analytically indicated that modulus of elasticity of alkali silica gel had a large effect on the cracking pattern and the amount of swelling. Especially, the modulus of elasticity less than that of normal concrete affected the analytical results very much.

(3) Mitigation effect of stirrups against ASR expansion was analytically evaluated. The analyses pointed out that the arrangement of stirrups in plural directions might have ASR expansion decrease more effectively than that in singular direction.

(4) While stirrups have effect on decrease in number of cracks, the width of crack might be increasing.

7 REFERENCES

[1] Kojima, T, and Mizuta, M (2006) : Numerical approach of crack propagation of RC structures using discrete crack model, Proceedings of The Kenji Sakata Symposium, 7th CANMET/ACI International conference : 229-243

[2] Matsuo, M, Hibino, K, Takagi, N, and Kojima, T (2000) : Discrete crack model for cracking in concrete and its fundamental characteristics, Journal of Materials, Concrete Structures and Pavements, Japan Society for Civil Engineers (No.655/V-48) : 1-12

Table 1: Values of [C].

C_{nn} (N/mm^3)	C_{tt} (N/mm^3)	C_{tn} (N/mm^3)	C_{nt} (N/mm^3)
variable (refer to Figure 2)	G/t	0	0

(G: shear modulus of rigidity, t: thickness of ASR element)

Table 2: Mechanical properties of mortar matrix and aggregate elements.

element	f'_c (N/mm^2)	f_t (N/mm^2)	E (N/mm^2)	ν
mortar matrix	25	2.8	2.8×10^4	0.1667
aggregate	25	2.8	5.6×10^4	0.1667

(f'_c: compressive strength, f_t: tensile strength, E: modulus of elasticity, ν : Poisson ratio)

Table 3: Modulus of elasticity for discrete crack elements.

mortar crack (N/mm^2)	aggregate crack (N/mm^2)	ASR ring (N/mm^2)
$10.0\times E_{c0}$	$20.0\times E_{c0}$	0.1 to $20.0\times E_{c0}$ (without stirrups) $1.0\times E_{c0}$ (with stirrups)

($E_{c0}=3.0\times10^4$ (N/mm^2))

Table 4: Analytical factors.

(a) Without stirrups

	modulus of elasticity of ASR ring element, Er (N/mm^2)		
case A	$0.5\times E_{c0}$	$1.0\times E_{c0}$	$10.0\times E_{c0}$
case B	0.1 to $20.0\times E_{c0}$		

(b) With stirrups

	arrangement of stirrups		diameter
	x-direction	y-direction	
case I	Yes	No	D6
case II	Yes	Yes	
case III	Yes	No	D13
case IV	No stirrups		

Table 5: Mechanical properties of stirrups.

	A_s (mm^2)	f_y (N/mm^2)	E_s (N/mm^2)
D6	31.67	295	2.0×10^5
D13	126.7		

(A_s: nominal cross sectional area, f_y: yield strength, E_s: modulus of elasticity)

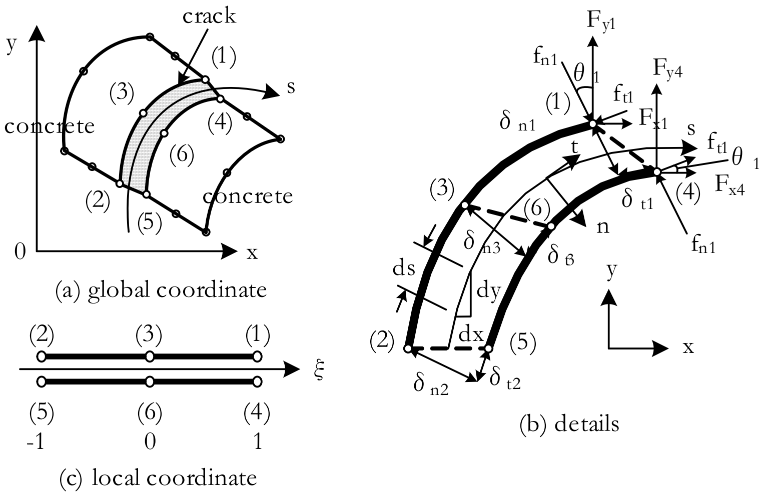


Figure 1 Modeling of discrete crack element

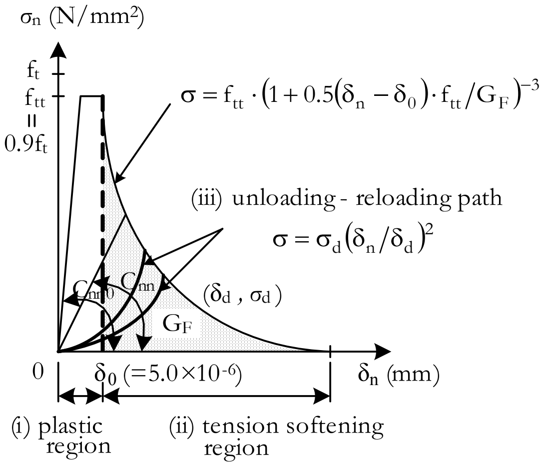


Figure 2 Transferring stress, σ_n − crack width, δ_n relationship

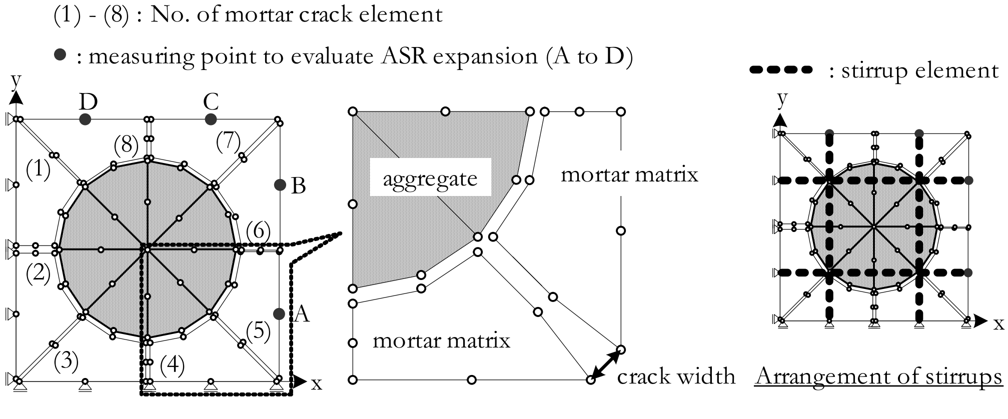


Figure 3 Modeling of ASR element

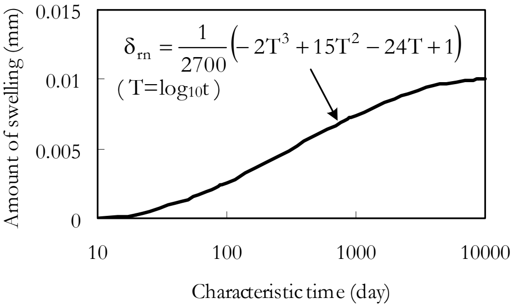


Figure 4 Amount of swelling, δ_{rn} - characteristic time, t relationship

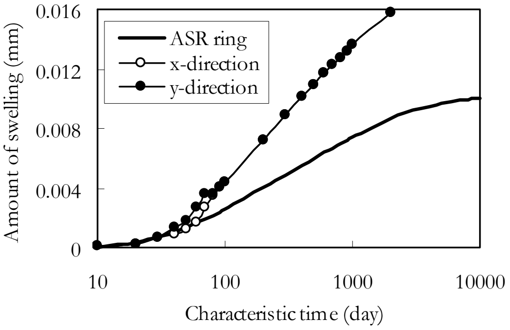


Figure 5 Amount of swelling - characteristic time relationship in case of $E_r=1.0\times E_{c0}$

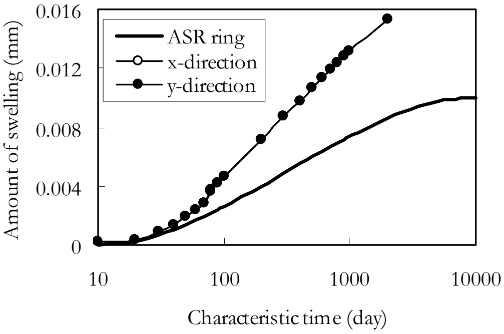


Figure 6 Amount of swelling - characteristic time relationship in case of $E_r=10.0\times E_{c0}$

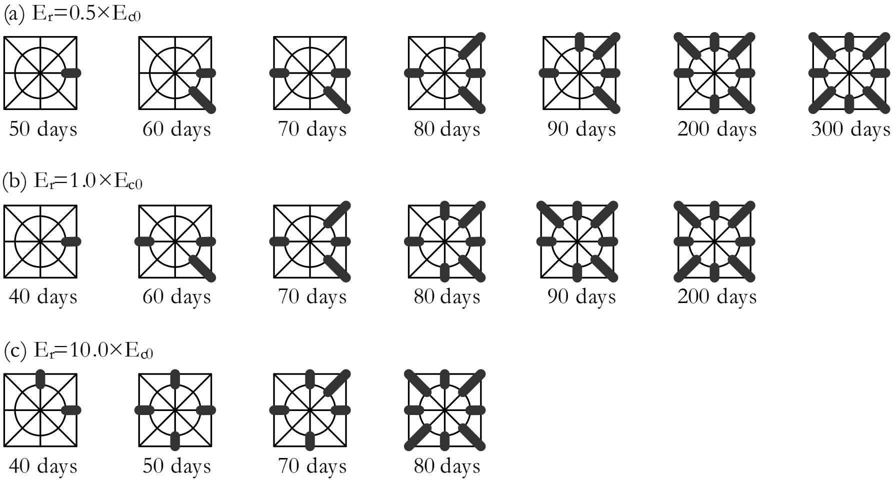
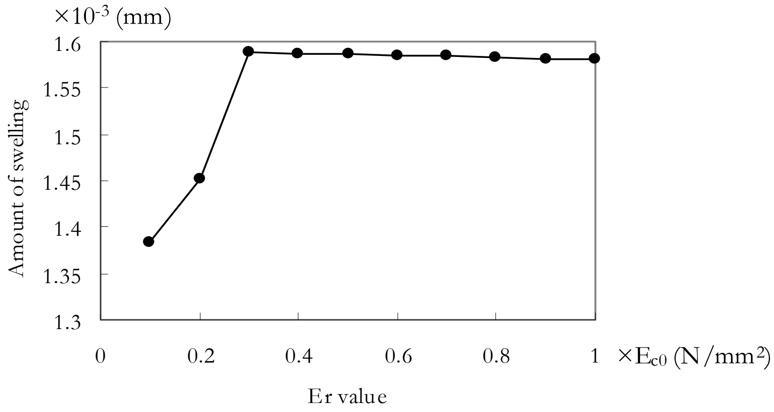
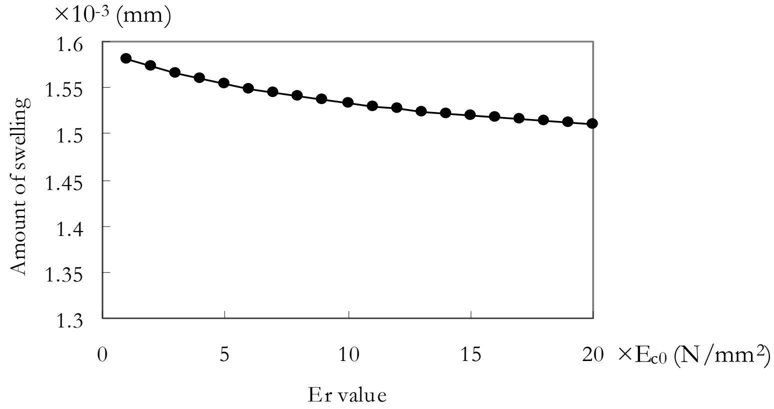


Figure 7 Cracking patterns



(a) Er ≤1



(b) Er ≥1

Figure 8 Amount of swelling－E_r value relationship

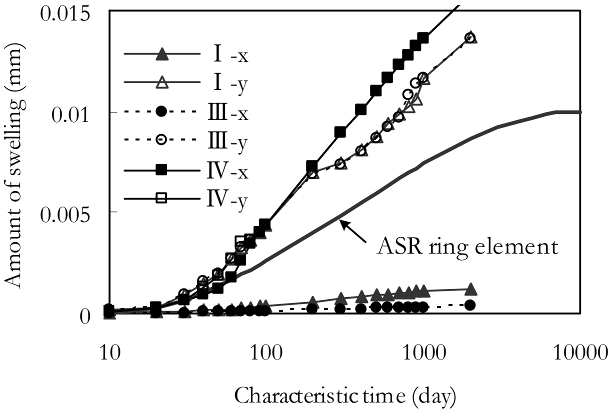
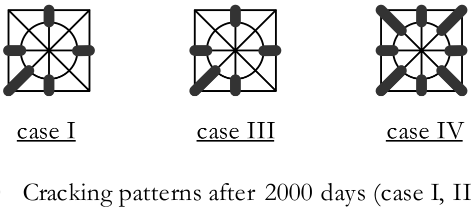


Figure 9 Amount of swelling - characteristic time relationship (case I, III and IV)



case I case III case IV

Figure 10 Cracking patterns after 2000 days (case I, III and IV)

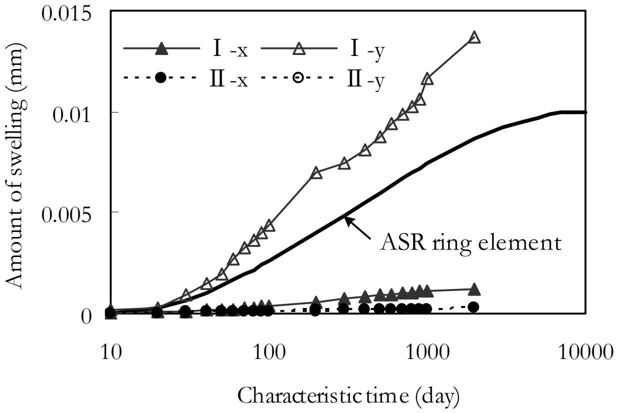


Figure 11 Amount of swelling - characteristic time relationship (case I and II)

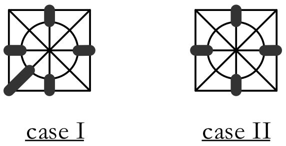


Figure 12 Cracking patterns after 2000 days (case I and II)